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HARVESTING MODELS WITH ALLEE EFFECTS IN RANDOMLY VARYING ENVIRONMENTS

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In a randomly varying environment, a harvesting model assumes the form of a stochastic differential equation (SDE)

$$dX(t) = f(X(t))X(t)dt + \sigma X(t)dW(t) - qE(t)X(t)dt, \quad X(0) = x,$$

where $X(t)$ is the harvested population size at time t, $f(X)$ is its mean (*per capita*) natural growth rate when its size is X (assumed to be of class C^1 and such that $f(0^+)$ is finite $\neq 0$ and $f(+\infty) < 0$), $\sigma dW(t)/dt$ describes the effect of environmental fluctuations on the growth rate (with $W(t)$ a standard Wiener process and $\sigma > 0$ a noise intensity parameter), $E(t)$ is the harvesting effort at time t and $q > 0$ the catchability. The yield per unit time is $H(t) = qE(t)X(t)$.

The profit per unit time is $\Pi(t) = P(t) - C(t)$, where $P(t) = p_1H(t) - p_2H^2(t)$ ($p_1 > 0$, $p_2 \geq 0$) is the sale price and $C(t) = c_1E(t) + c_2E^2(t)$ ($c_1,\ c_2 > 0$) is the cost.

We consider the case of no Allee effects (f strictly decreasing with $f(0^+) > 0$) and the case of Allee effects (there are constants $0 < L < K$ such that $f(K) = 0$, f increases strictly for $0 < X > L$ and strictly decreases for $X > L$). Allee effects ([1]) may occur at low population sizes ($0 < X < L$) when, for instance, the geographical dispersion makes if difficult for individuals to find mating partners or to mount an effective collective defence against predators.

In the absence of fishing, we know that (see [4] and [6]), "mathematical" extinction ($X(t) \to 0$ as $t \to +\infty$) will occur a.s. if there are strong Allee effects ($f(0^+) < 0$) and will a.s. not occur if there are weak Allee effects ($f(0^+) > 0$). Here, we study optimal sustainable harvesting policies with constant harvesting effort ($E(t) \equiv E$), determining the conditions for non-extinction and existence of a stationary density and obtaining, under such conditions, the constant effort E^{**} that maximizes the expected profit per unit time at the stationary regimen (following the ideas in [2] and [3]).

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We then look at the particular case of the logistic-like model with Allee effects $f(X) = r\left(1-\frac{X}{K}\right)\left(\frac{X-A}{K-A}\right)$, which provides weak Allee effects when $-K < A < 0$, strong Allee effects when $0 < A < K$ and retrieves the logistic model when $A \to -\infty$.

For that particular model with weak Allee effects, following the ideas used in [5] (where the no Allee effects case was studied) and using real fishery data, we compare the results obtained under the optimal constant effort sustainable harvesting policy with those obtained by the optimal variable effort $E^*(t)$ policy, that is the policy that maximizes the expected discounted profit over a time horizon T (using optimal control theory). The $E^*(t)$ policy was studied using numerical techniques developed in [7]. The implementation of this policy requires constant knowledge of the population size (an inaccurate, costly and difficult process) and leads to wildly varying efforts and heavy social implications, being inapplicable in practice. As we will see, the E^{**} policy is less profitable but does not have these disadvantages and is very simple to apply. We will also study the effect of Allee effects by comparing the logistic model without Allee effects with the logistic-like model with weak Allee effects (considering different values of parameter A).

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