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ALLEE'S EFFECT BIFURCATION IN A 2D EXPONENTIAL DIFFEOMORPHISM

J. Leonel Rocha[1]*, A-K. Taha[2] and D. Fournier-Prunaret[3]

[1]CEAUL, ADM, ISEL-Engineering Superior Institute of Lisbon, Rua Conselheiro Emídio Navarro 1, 1959-007 Lisboa, Portugal

[2]INSA Toulouse, Federal University of Toulouse Midi-Pyrénées, 135 Avenue de Rangueil, 31077 Toulouse, France

[3]LAAS-CNRS, INSA Toulouse, Federal University of Toulouse Midi-Pyrénées, 7 Avenue du Colonel Roche, 31077 Toulouse, France

jrocha@adm.isel.pt (*corresponding author), taha@insa-toulouse.fr, daniele.fournier@insa-toulouse.fr

The main purpose of this talk is to present the fundamentals of the dynamics and bifurcations of an embedding of one-dimensional generic growth functions into a two-dimensional diffeomorphism, $T_b : \mathbb{R}^2 \to \mathbb{R}^2$, which is defined in the form of recurrence relationship as follows,

$$T_b \equiv \begin{cases} x_{n+1} = f(x_n; \beta, \gamma, r) + y_n \\ y_{n+1} = bx_n \end{cases} \Leftrightarrow T_b \equiv \begin{cases} f_1(x_n, y_n; \beta, \gamma, r) = rx_n^{1+\beta(1-\gamma)} \left(1 - x_n^{\beta}\right)^{\gamma} + y_n \\ f_2(x_n, y_n; \beta, \gamma, r) = bx_n \end{cases},$$

where $0 \leq b \leq 1$ is the embedding parameter, $(x_n, y_n) \in [0, 1] \times [0, 1]$, $n \in \mathbb{N}$, and has constant Jacobian determinant $J = -b$. This planar map T_b is defined in a parameters space

$$\Sigma_b = \left\{ (\beta, \gamma, r, b) \in \mathbb{R}^4 : \gamma < 1 + \frac{1}{\beta},\ 0 \leq b \leq 1, \text{ with } \beta, \gamma, r > 0 \right\}.$$

From the point of view of ecological and biological research, this diffeomorphism is related to the population size evolution of two species using the generalized logistic growth equation in one of the species and naturally incorporates a key topic in these research areas: the Allee effect. Consequently, the presence of this species extinction phenomenon leads us to a new definition of bifurcation: the Allee effect bifurcation. The stability and the nature of the fixed points of the two-dimensional diffeomorphism are analyzed, by studying the corresponding contour lines. Fold and flip bifurcation structures of this exponential diffeomorphism are investigated, in which there exist flip codimension-2 bifurcation points and cusp points, when some parameters evolve. Analytical results will be illustrated with numerical simulations and appropriate bifurcation diagrams.

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