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INDEFINITE NONLINEAR WEIGHT PROBLEMS IN POPULATION GENETICS

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We study Neumann BVPs associated with $u'' + w(x)f(u) = 0$, where $\mathcal{I}$ is a bounded interval, the weight $w\colon \mathcal{I} \to \mathbb{R}$ is sign-changing and the nonlinearity $f\colon [0,1] \to \mathbb{R}$ satisfies $f(0) = f(1) = 0$, $f(s) > 0$, $\forall\, s \in]0,1[$. Looking first at the graph of f and then at the shape of w, we deal with the multiplicity of nontrivial positive solutions to such kind of problems. Firstly, we answer a conjecture appeared in the field of population genetics in [3, 4] that states whether a uniqueness result of positive solutions holds if $\int_{\mathcal{I}} w < 0$, f is not concave and $s \mapsto f(s)/s$ is decreasing. Indeed, we show the existence of at least 3 nontrivial positive solutions by considering a function f which fulfills the conjecture's conditions and has a strict local minimum in $]0,1[$ (see [6]). Secondly, we focus on the weight term and, compared with [1, 5], we prove the existence of at least 8 nontrivial positive solutions if w has two positive humps separated by a negative one and $f'(0) = 0$ (see [2]).

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