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NON-LINEAR STOCHASTIC PREDATOR-PREY POPULATION MODELS WITH MASS CONSERVATION

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A classical predator-prey model is used to compare dynamics behaviour of a deterministic and stochastic predator prey model formulation. We will analyse two deterministic predator-prey models. With the Rosenzweig-MacAthur model the environmental resources for the prey are not explicitly modelled. In the alternative mass balance model these resources are modelled in a spatially homogeneous environment for which batch conditions are assumed. Assuming remineralisation of excreted products by the two populations (including minerals and dead material) biomass conservation is guarantied. Then the law of mass conservation is used to reduce this model to a system of two ODEs.

The predator-prey trophic interaction is generally modelled by the Holling type II functional response [1]. With the derivation of the algebraic expression for the ingestion rate of the predator consuming the prey a time-scale argument is used [2, 4] whereby the predator population is split up into searchers and handlers giving a three dimensional ODE system. However, it appears that in this formulation the same terms appear for both the prey and searching predator dynamics description while their dynamics run at different time-scales. To solve this inconsistency we use besides the mass based also a number based model formulation. In this talk this argument is derived rigorously using predator/prey body-size ratios based on experimental data. This same formulation is used in the context of a stochastic model formulation. The deterministic models are analysed using bifurcation analysis and the stochastic model by realisations following the direct Gillespie method [3]. The dynamics predicted by the various formulations are compared with a dimensionless version of

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the predator-prey model.

There are two types of convergences with respect to the size of the prey and predator individuals. First for the deterministic versions, if the ratio of the predator/prey body sizes becomes large the dynamics of three dimensional system approximates that for the two dimensional original predator-prey model. This is the classical situation where the dimension of the system is reduced using a time-scale argument. When furthermore the size of the prey individuals become small the large number assumption holds and the dynamics for the stochastic version three dimensional system also approximates that for the deterministic two dimensional predator-prey models.

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