CLASSIFICATION OF SPATIAL PATTERNS ARISING IN SPATIO – TEMPORAL DYNAMICS OF INVASIVE SPECIES

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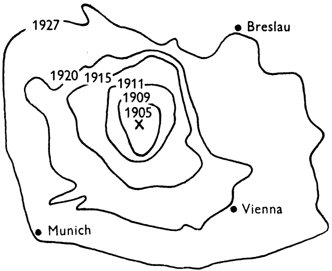
Outline

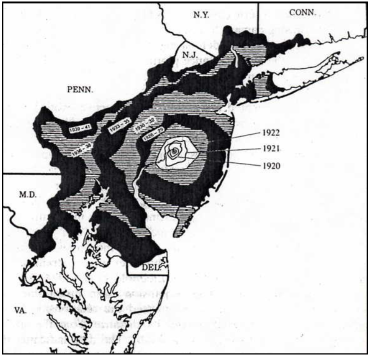
- Introduction: spread of invasive species into the space, continuous front vs. patchy invasion
- Modelling biological invasion
- Study of spatial patterns:
  - Classification of spatial patterns in the mathematical model
  - Can we distinguish continuous front from patchy invasion spatial patterns?
  - How robust are spatial patterns?
- Conclusions

Introduction

Examples of biological invasion

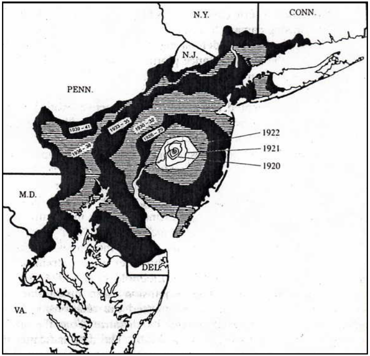


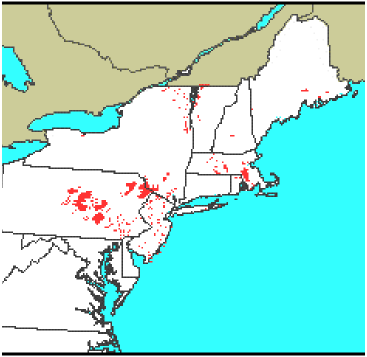
Invasion of muskrats
(*Ondatra zibethica*)
in central Europe



Invasion of Japanese beetle
(*Popillia japonica*)
in the United States

Is it always 2-D traveling front?





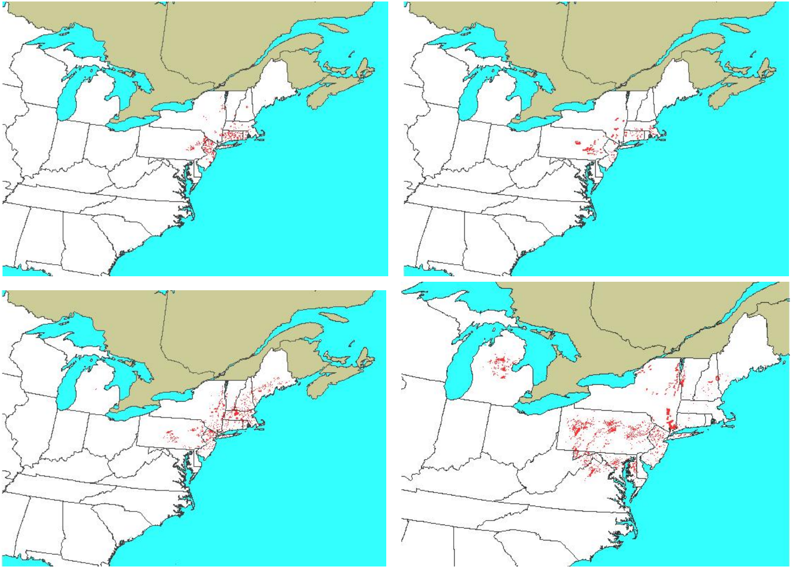
Invasion of Japanese beetle
(*Popillia japonica*)
in the United States

2-D traveling front

Invasion of Gypsy moth
(*Lymantria dispar*)
in the United States

Patchy invasion

Geographic spread of Gypsy moth



(by courtesy of Andrew Liebhold)

Modelling biological invasion

The IDE-based framework

We consider a system of integro-difference equations:

$$\begin{aligned} u_{t+1}(\mathbf{r}) &= \int_\Omega k^{(u)}\left(|\mathbf{r}-\mathbf{r}'|\right) f\left(u_t\left(\mathbf{r}'\right), v_t\left(\mathbf{r}'\right)\right) d\mathbf{r}', \\ v_{t+1}(\mathbf{r}) &= \int_\Omega k^{(v)}\left(|\mathbf{r}-\mathbf{r}'|\right) g\left(u_t\left(\mathbf{r}'\right), v_t\left(\mathbf{r}'\right)\right) d\mathbf{r}', \end{aligned}$$

- The dispersal kernel $k\left(|\mathbf{r}-\mathbf{r}'|\right)$ gives the probability density of the event that an individual located at the position $\mathbf{r}'$ before the dispersal will be found at the position $\mathbf{r}$ after the dispersal.
- The Gaussian kernel

$$k_G\left(|\mathbf{r}-\mathbf{r}'|\right) = \frac{1}{2\pi\alpha_i^2}\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{2\alpha_i^2}\right).$$

The IDE-based framework

We assume that both species have a similar life cycle so that they interact during their maturation stage:

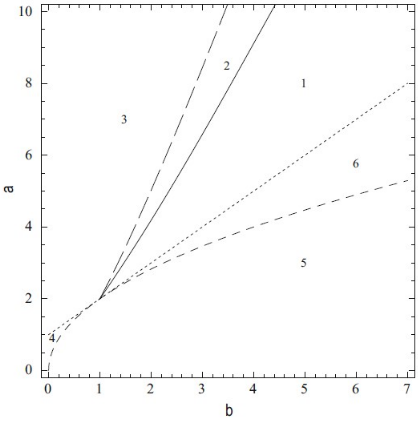
$$\tilde{u}_t(\mathbf{r}) \; = \; f\left(u_t\left(\mathbf{r}\right), v_t\left(\mathbf{r}\right)\right), \qquad \tilde{v}_t(\mathbf{r}) \; = \; g\left(u_t\left(\mathbf{r}\right), v_t\left(\mathbf{r}\right)\right),$$

where $\tilde{u}_t(\mathbf{r})$ and $\tilde{v}_t(\mathbf{r})$ are the population densities prior the dispersal stage,

$$\begin{aligned} f(u,v) &= \frac{a\left(u(\mathbf{r})\right)^2}{1 + b\left(u(\mathbf{r})\right)^2} \cdot \exp\left(-v(\mathbf{r})\right), \\ g(u,v) &= u(\mathbf{r})v(\mathbf{r}), \end{aligned}$$

$a = A/\delta$, $b = (B/\delta)^2$, A is the prey intrinsic growth rate, $1/B$ is the prey density for which its per capita growth rate reaches its maximum, and δ is the predator growth rate.

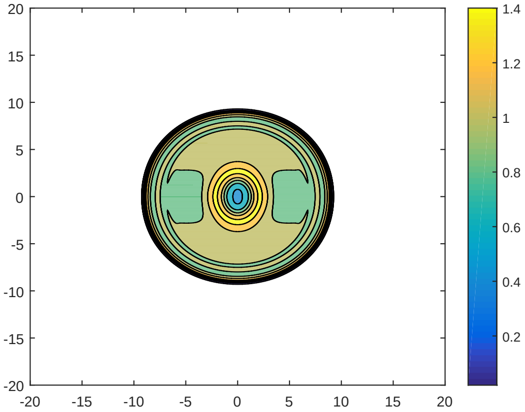
The parametric plane



L.A.D. Rodrigues, D.C.Mistro, S.V.Petrovskii (2012) Pattern formation in a space- and time-discrete predator-prey system with a strong Allee effect. Theor. Ecol. 5:341-362.

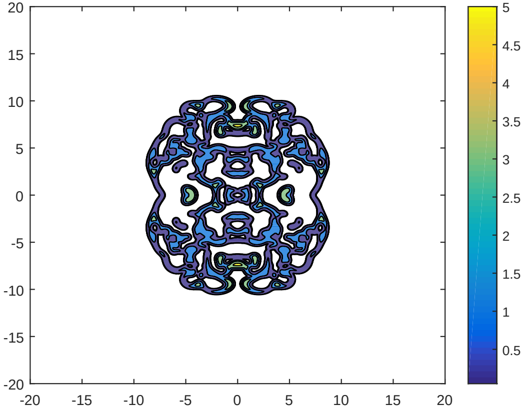
Solution in Domain 2 of the parametric plane

The temporal dynamics is oscillatory for any parameters from Domain 2. Topologically, the density distribution is a convex continuous front ($a = 4.0$ and $b = 1.8$; time $t = 200$).



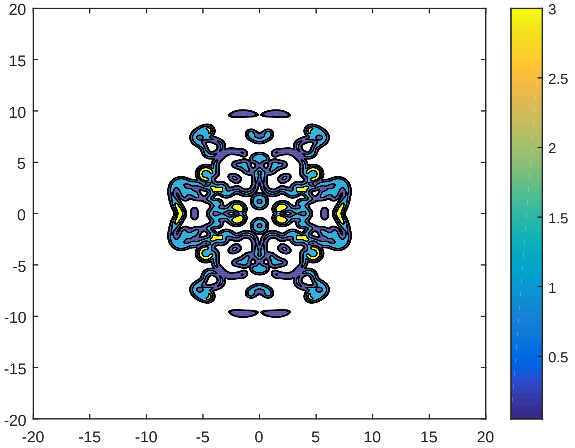
Solution in Domain 3 of the parametric plane

The pattern of spread depends on the sub-domain of Domain 3 where the parameters are taken. A concave continuous front obtained for $a = 4.0$ and $b = 0.716$; time $t = 200$.



Solution in Domain 3 of the parametric plane

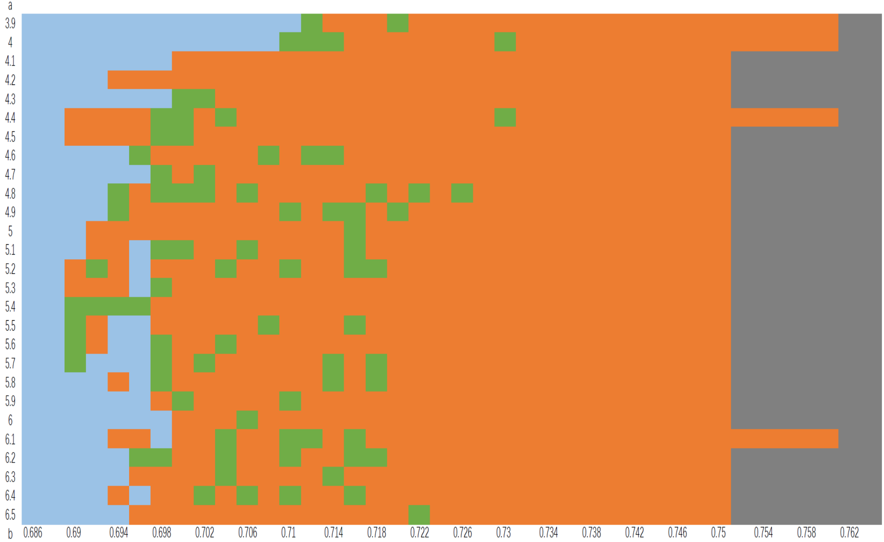
Patchy spread (without any continuous front) obtained at time $t = 200$ for parameters $a = 4.0$ and $b = 0.714$ taken from Domain 3.



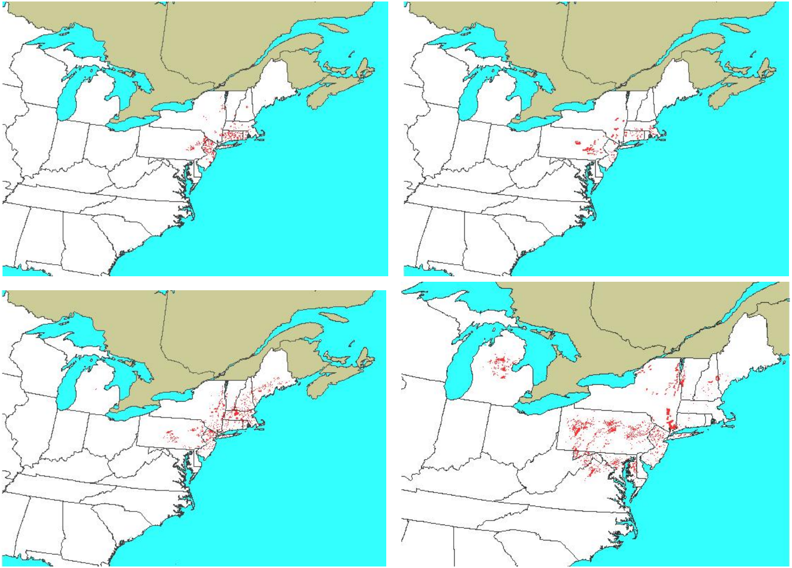
Detection and classification of spatial patterns

Spatial patterns in the parametric plane (Domain 3)

blue – the extinction, orange – a concave continuous front,
grey – a convex continuous front, green – the patchy invasion



Real-life ecological data: patchy or continuous?



(by courtesy of Andrew Liebhold)

Accuracy of real-life ecological data

- Very small values of the population density are often impossible to detect due to limitations of sampling/monitoring techniques, the minimum detectable density being called the ‘detection threshold’.

- How much information about the population density is detected depends also on the number of sampling locations used in a monitoring routine.

- Correspondingly, we investigate the sensitivity of the spatial pattern to the cut-off parameter C and the number of sampling locations N.

The cut-off parameter

- The density $u(x, y) \neq 0$ at any point (x, y).

- Small values of the population density are ignored in the model and in real-life monitoring.

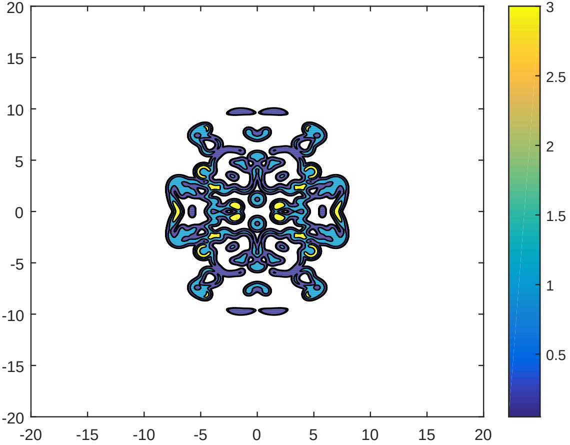
- Cut-off parameter C

$$\hat{u}(x, y) = u(x, y) \text{ for } u(x, y) > C,$$

$$\hat{u}(x, y) = 0 \text{ for } u(x, y) \leq C.$$

Patchy density distribution from a mathematical model

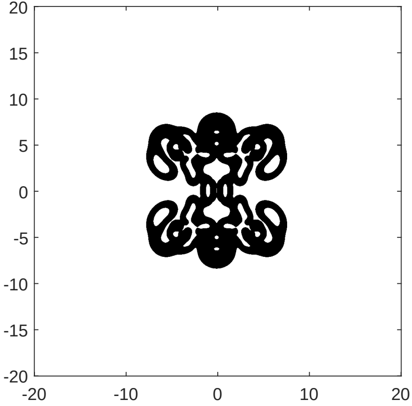
Cut-off, C: 1% of the max density value,
Grid, N: 1025 $\times$ 1025 points

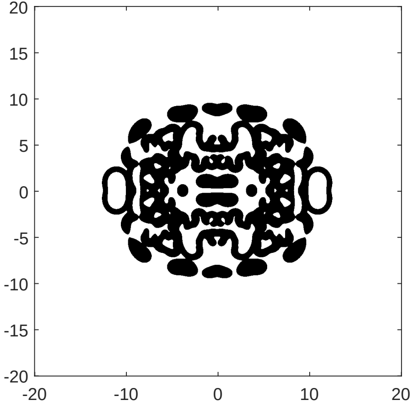


Binary images

Binary presence/absence maps are defined as

$$\hat{u}(x,y) = 1 \quad \text{for} \quad u(x,y) > C, \quad \hat{u}(x,y) = 0 \quad \text{for} \quad u(x,y) \leq C.$$





The number of objects

- We use the Image Processing Toolbox in MATLAB to count number n of separate patches for a given value of cut-off C.
- We ignore a complex topological structure of the spatial density distribution within any sub-domain of the non-zero density with a closed boundary (e.g. density patterns behind a continuous front)
- A concave front then counts as a single object.
- A sufficiently large increase in the cut-off C should break the single object into a collection of multiple objects.

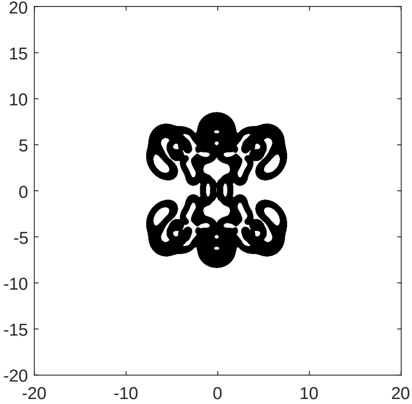
Transformation of a continuous front distribution

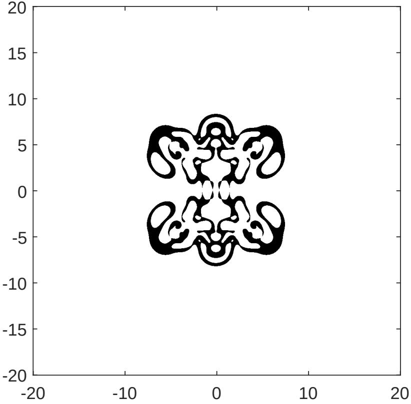
The number of disconnected objects n for the concave-front pattern when cut-off value C is varied with an increment of 0.001.

C	0.05	$\cdots$	0.826	$\cdots$	0.858	$\cdots$	0.879	...	0.905
n	1	1	5	5	6	6	10	10	14

Example: decomposition of a concave front

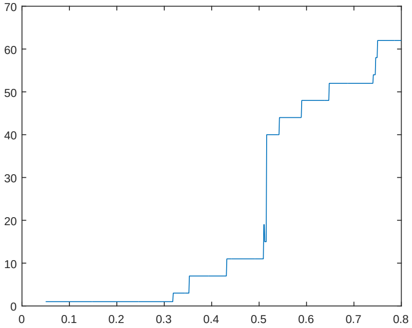
A concave continuous front pattern is transformed into a patchy distribution consisting of six disconnected patches at $C = 0.860$

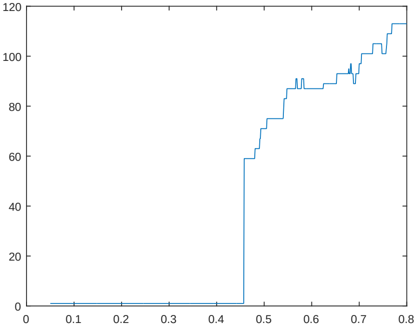




Transformation of a continuous front distribution

The number of objects as a function of the cut-off value C for two concave-front distributions. Parameters are $a = 6.0$, $b = 0.710$ and $a = 5.7$, $b = 0.711$.





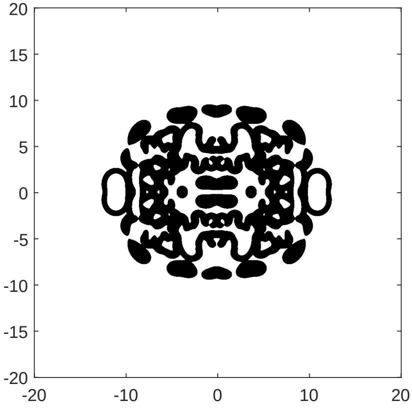
Transformation of a patchy distribution

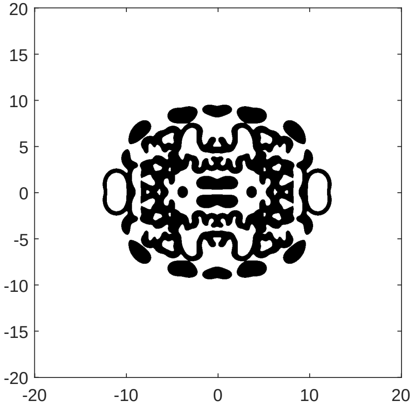
The number of disconnected objects n for the patchy pattern when cut-off value C is varied with an increment of 0.001.

C	0.05	$\cdots$	0.065	$\cdots$	0.108	$\cdots$	0.157	$\cdots$	0.540
n	3	3	5	5	9	9	13	13	15

Example: decomposition of a patchy pattern

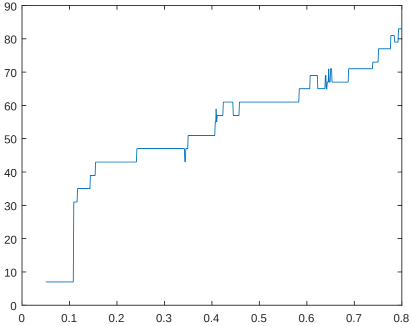
Transition from five to nine separate patches happens at $C = 0.130$.

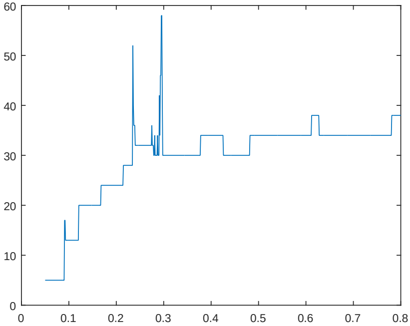




Transformation of a patchy distribution

The number of objects as a function of the cut-off value C for two patchy distributions. Parameters are $a = 5.2$, $b = 0.71$ and $a = 5.9$ and $b = 0.71$.

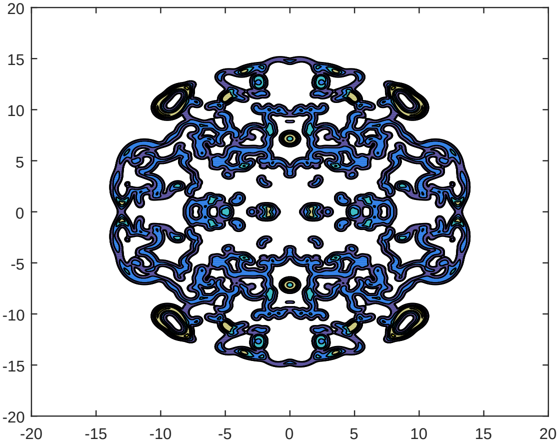




A concave distribution on a fine computational grid

$$a = 6.0, \;\; b = 0.71$$

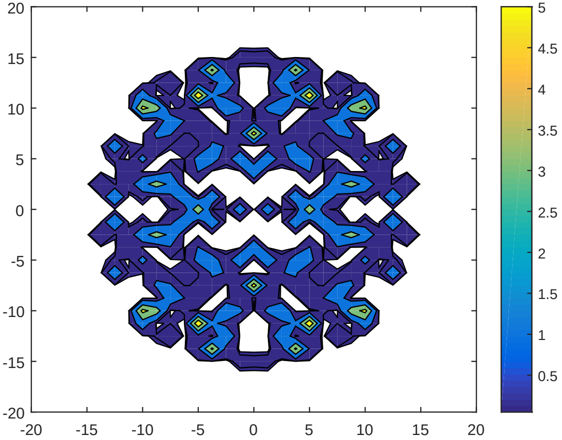
The density distribution as a solution to the IDE problem:
grid is 1025×1025 points



A concave distribution on a coarse sampling grid

$$a = 6.0, \;\; b = 0.71$$

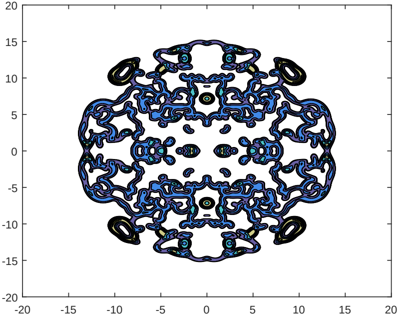
Collecting information about the density distribution:
grid is 33×33 points



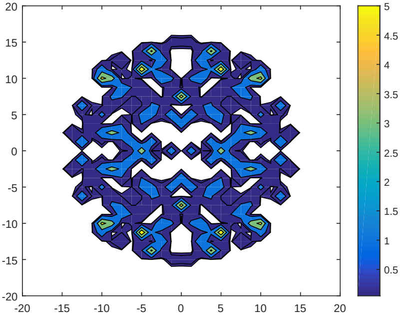
A concave distribution on a fine and coarse grid

$a = 6.0, \ \ b = 0.71$

grid is 1025×1025 points



grid is 33×33 points



Transformation of the concave distribution

- Generate the binary map of the density distribution on a coarse grid.
- Compute the number of disconnected objects on the coarse grid.
- Decrease the number of grid points, restore the density distribution (i.e. take data from the original fine grid) and repeat the above.

Transformation of the concave distribution

The number of disconnected objects n for the concave pattern when the number N of grid points decreases

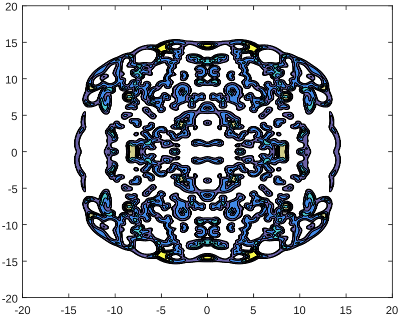
N	1025	513	257	129	65	33	17	9	5	3
n	1	1	1	1	1	1	1	4	4	0

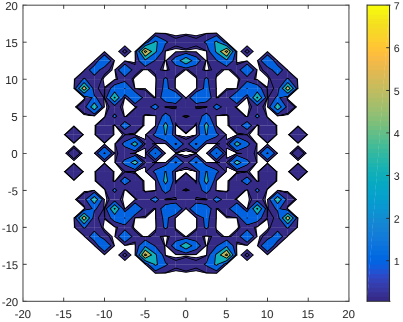
A patchy distribution on a fine and coarse grid

$$a = 6.4, \;\; b = 0.71$$

grid is 1025×1025 points

grid is 33×33 points





Transformation of the patchy distribution

The number of disconnected objects *n* for the patchy pattern when the number *N* of grid points decreases

N	1025	513	257	129	65	33	17	9	5	3
n	35	33	29	17	13	13	4	4	2	0

Conclusions

Both continuous front and patchy spatial patterns remain robust to the factors that can affect the results of monitoring.

- Patchy invasion is not an artefact of a poor monitoring protocol.
- Future study is required to identify the factors affecting the spatial pattern of invasive spread in the context of the invasive species monitoring.

References

L.A.D.Rodrigues, D.C.Mistro, E.R.Cara, N.B.Petrovskaya, S.V.Petrovskii. *Patchy Invasion of Stage-Structured Alien Species with Short-Distance and Long-Distance Dispersal.* Bull. Math. Biol. (2015) 77:1583-1619

N.B.Petrovskaya, S.V. Petrovskii, W. Zhang. *Patchy, not Patchy, or How Much Patchy? Classification of Spatial Patterns Appearing in a Model of Biological Invasion.* Math. Model. Nat. Phenom. (2017) 12:208-225